

QUASISTATIC EVOLUTION OF MAGNETOSTATIC CORONAL STRUCTURES

Richard Wolfson
 Department of Physics
 Middlebury College
 Middlebury, VT 05753

INTRODUCTION

This paper reviews four separate but related studies of coronal magnetostatic equilibria under a variety of boundary conditions and distributions of coronal current. Physically, all four studies assume an axisymmetric corona whose radial magnetic field at the coronal base is dipolar. Electric currents in the model coronas are assumed to flow in the azimuthal direction, giving rise to Lorentz forces that must be balanced by pressure gradients and gravity. Mathematically, such coronas are described by the equation of magnetostatic force balance and Ampere's law; under the model assumptions these equations combine to give a single equation for the effective potential $AR\sin\theta$:

$$\frac{1}{\sin\theta} \frac{\partial^2 (AR\sin\theta)}{\partial R^2} + \frac{\partial}{\partial \theta} \frac{1}{R^2 \sin\theta} \frac{\partial (AR\sin\theta)}{\partial \theta} = -4\pi R \sin\theta e^{-\int [g(R)/C^2] dR} F(AR\sin\theta) \quad (1)$$

(Hundhausen, Hundhausen, and Zweibel 1981; hereafter HHZ), where F is a free function. The case $F=0$ corresponds to a potential magnetic field decoupled from an atmosphere whose pressure varies only with radius. HHZ obtained analytic solutions to the linear problem arising when the function F is a constant. This case corresponds to a corona inflated with excess material throughout its entire volume, with pressure varying from pole to equator at any fixed radius. The studies reported here involve nonlinear functions $F(AR\sin\theta)$ and/or more elaborate boundary conditions, with the aim of providing more realistic models for observed coronal structures. Since all the models contain adjustable parameters related to the amount of excess mass loading in the corona, variation of these parameters can be used to study the quasistatic evolution of a model corona in response to mass addition.

CURRENT SHEET BOUNDARY CONDITIONS

The solar wind is a dominant influence on the coronal magnetic field, drawing some field lines into open configurations where they merge with the interplanetary field. Describing this situation in the context of a static model requires an artificial boundary condition that forces field lines to open. A common approach employs a source surface, beyond which the field lines are assumed radial (Altschuler and Newkirk 1969). In the present model, we assume instead that a current sheet exists in the equatorial plane, beginning at some height R_1 and extending to infinity. This approach models more realistically the observed behavior of the solar magnetic field in crossing the equatorial

plane. Mathematically, the current sheet is incorporated by forcing the field component B_θ to zero for points in the equatorial plane with $R > R_1$. With this boundary condition, and with the function F in equation (1) being a constant, the solution may be written in terms of Legendre polynomials (Wolfson 1985). Increasing the constant value of the function F gives a series of model coronas that deviate increasingly from the potential field corona (Fig. 1).

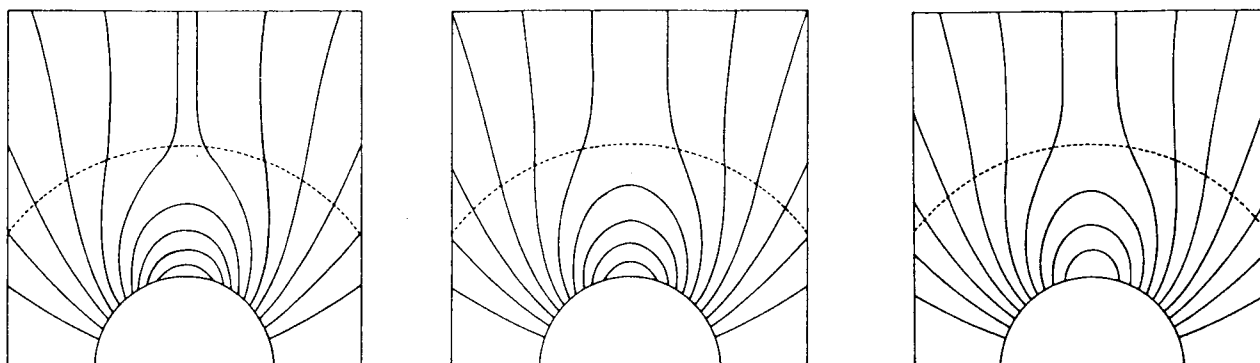


Fig. 1: Field lines for model coronas with increasing excess mass (from Wolfson 1985).

Analysis of the coronal current distribution in the open-field region shows that the current density decreases slightly from pole to equator at a fixed radius, then undergoes an abrupt reversal at the current sheet. This feature, along with flexibility in treating nonpotential fields, makes the present model a more realistic approach than the spherical source surface model.

A LIMITED CORONAL INFLATION

Choosing the function F in equation (1) to be a step function gives a model corona inflated with excess mass only below a particular field line. Fig. 2 shows the position of the field line that bounds the inflated region, for a variety of inflation parameters.

An interesting feature of the nonlinear model arising from a step function F is that solutions cease to exist, or undergo a dramatic and discontinuous change in morphology, above a certain value of the inflation parameter. We have suggested that this lack of nearby equilibrium solutions corresponds to the onset of dynamical processes which our quasistatic model cannot describe. For realistic coronal parameters, the mass associated with this loss of equilibrium is on the order of 10^{16} g--comparable to the mass ejected in a coronal mass ejection (CME).

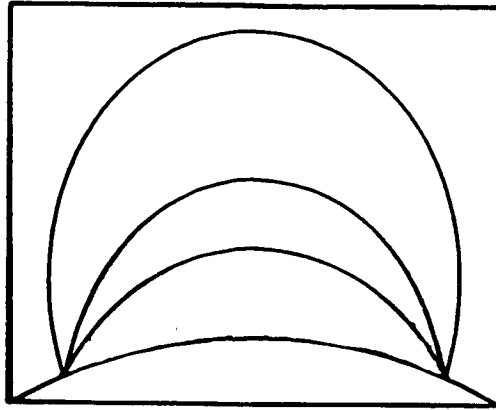


Fig. 2: Expansion of the field line bounding a region of inflated corona. Lowest field line shown is that of a potential field.

THE INFLATED CORONAL FLUX TUBE

Specifying the function F to be zero everywhere except for a limited region where it is a bipolar function gives a model corona whose magnetic field is everywhere potential except for a narrow tube (actually a torus in the axisymmetric geometry), which constitutes a high-pressure, high-density region surrounded by lower pressure corona. The resulting model may then provide a description of a white-light coronal loop. Increasing the amplitude of the function F corresponds to loading more mass into the tube. Fig. 3 shows that the result is an increase in the width of the tube coming almost entirely from an expansion of its outer bounding field line.

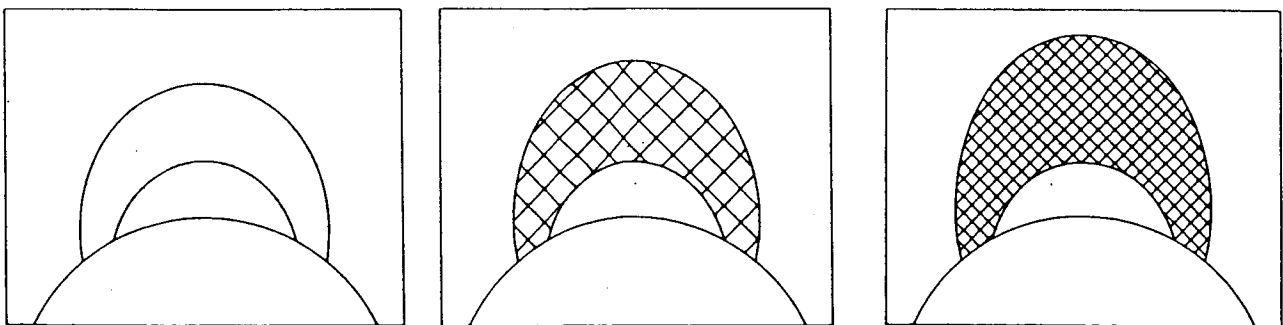


Fig. 3: Changing configuration of a "flux tube" as excess mass is added between specified field lines. Hatching indicates relative amount of excess mass (from Wolfson and Conover 1986).

That the tube top rises instead of falling as mass is added is a result of the increased Lorentz forces associated with the increasing coronal currents.

This nonlinear model exhibits the same loss of equilibrium as the previously described model; again, the excess mass involved is on the order of that ejected in a CME.

QUASISTATIC EVOLUTION OF A CORONAL STREAMER

The final and most ambitious calculation in this series attempts to model the quasistatic evolution of a coronal streamer prior to its disruption by an eruptive prominence and associated coronal mass ejection. Such an evolution occurred over a two-day period in August, 1980, as the closed-field region of a large and well-defined helmet streamer appeared to bulge and brighten before being disrupted by the eruption of the underlying prominence on August 18 (Illing and Hundhausen 1986). The model calculation involves inflating the coronal region below a specified field line, while applying current-sheet boundary conditions above that line. Total flux in the closed-field region is held constant, and the top height of the closed region adjusted accordingly as excess mass is added. Fig. 4 shows a sequence of model equilibria, beginning with a best fit to the initial observations of the streamer and ending with the greatest value of coronal inflation for which equilibrium solutions can be found.

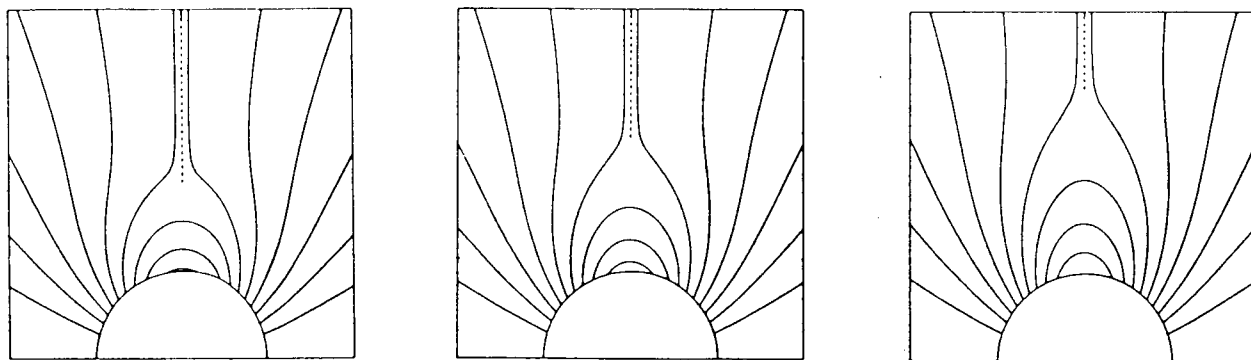


Fig. 4: A sequence of model equilibria describing the quasistatic evolution of a coronal helmet streamer (from Conover, Wolfson, and Illing, in preparation).

The calculated top heights and corresponding excess masses are consistent with measurements taken from coronagraph images (Conover, Wolfson and Illing, in preparation). Furthermore, the equilibrium sequence terminates at about the maximum top height and excess mass observed before the slow evolution gives way to the prominence eruption and mass ejection.

CONCLUSION

Although highly idealized, the axisymmetric magnetostatic models described in this paper provide insights into the behavior of a variety of coronal structures. The models show how more realistic current-sheet boundary conditions and coronal density variations may be modeled. And the nonlinear models invariably cease to have equilibrium solutions as the excess coronal mass is increased--just as the corona itself may undergo a transition to the dynamic behavior of a coronal mass ejection.

This work was supported by NSF grant AST-8311338 to Middlebury College.

REFERENCES

- Altschuler, M.D. and Newkirk, G. 1969, Solar Phys., 9, 131.
- Conover, C., Wolfson, R., and Illing, R.M.E. "The Evolution of a Coronal Helmet Streamer," in preparation.
- Hundhausen, J.R., Hundhausen, A.J., and Zweibel, E.G. 1981, J. Geophys. Res., 86, 11117.
- Illing, R.M.E. and Hundhausen, A.J. 1986, J. Geophys. Res., in press.
- Wolfson, R. 1985, Astroph. J., 288, 769.
- Wolfson, R. and Conover, C. 1985, "The Inflated Flux Tube: a Model for White-Light Coronal Loops," submitted.
- Wolfson, R. and Gould, S.A. 1985, Astroph. J., 296, 287.